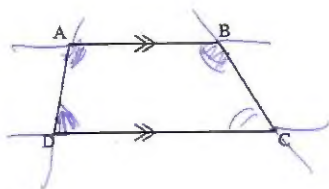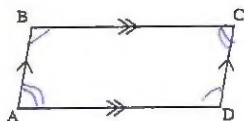## Special Quadrilaterals

Trapezium

- A trapezium has two parallel sides.

- The sum of the interior angles at the ends of each non-parallel side is 180°. That is,

  $\angle A + \angle D = 180°$ and $\angle B + \angle C = 180°$
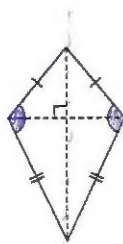
Parallelogram

- A parallelogram has opposite sides parallel.

- Its opposite sides are equal.
- Its diagonals bisect each other.
- Its opposite angles are equal. That is,

  $\angle A = \angle C$ and $\angle B = \angle D$

Rhombus

- A rhombus is a parallelogram with all its sides equal.

- Its diagonals bisect each other at right angles.
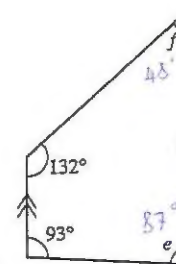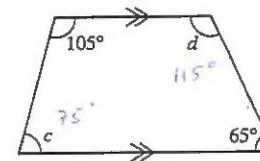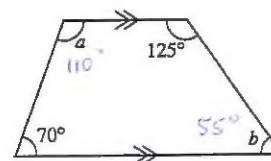- Its diagonals also bisect the angles at the vertices.
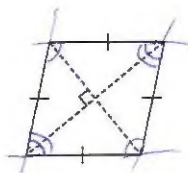
Kite

- A kite is a quadrilateral with two pairs of equal adjacent sides.

- Its longer diagonal bisects its shorter diagonal at right angles.
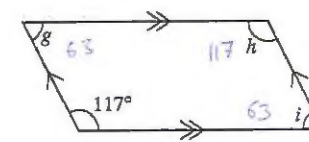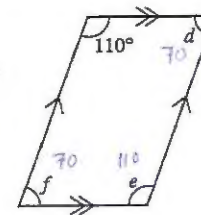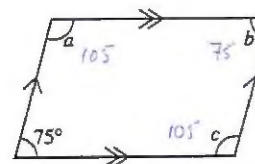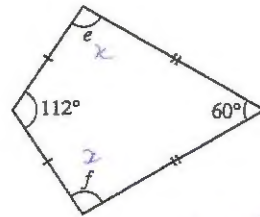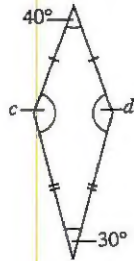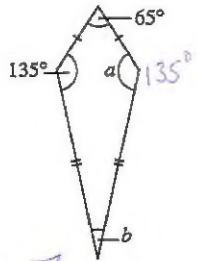- The opposite angles between the sides of different lengths are equal.

Exercise 5B

1 For each of these trapeziums, calculate the value of the lettered angles.

2 For each of these parallelograms, calculate the value of the lettered angles.

3 For each of these kites, calculate the value of the lettered angles.

65°
135° a $135°$

$b = 360 - (2 \times 135 + 65)$
$b = 25°$

40°
c d
30°

$c + d = 360 - (30 + 40)$
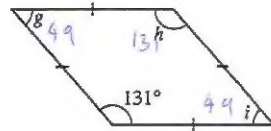$c + d = 290$
$2c = 290$
$c = 145$
$d = 145$

e
x
112°   60°
x
f

$2x = 360 - (112 + 60)$
$2x = 188$
$x = 94°$

4 For each of these rhombuses, calculate the value of the lettered angles.

a 105 75 b
75° 105 c

d 93 87 e
87° 93 f

g 49 131 h
131° 49 i

5 For each of these shapes, calculate the value of the lettered angles.

41°
122° 133°
a $58°$ $47°$ b

141° c 141
d

e 65
115° 115 f
65 g

$d = 360 - (2 \times 141 + 41)$
$d = 37°$

6 Calculate the values of x and y in each of these parallelograms.

a
$3x$
$4x + 5$ $5y$
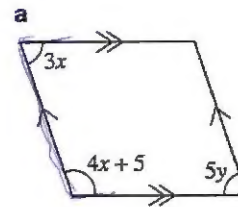
$3x + 4x + 5 = 180$
$7x = 175$
$x = 25°$

$4x + 5 + 5y = 180$
$100 + 5 + 5y = 180$
$5y = 75$
$y = 15°$

b
$11x + 7$ $5y$
$10x + 5$
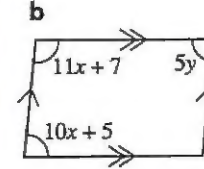
$21x + 12 = 180$
$x = 8°$

$5y = 10x + 5$
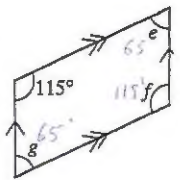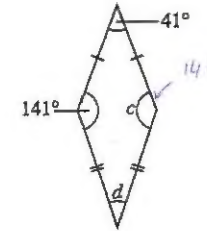$5y = 85$
$y = 17°$

c
$4y$
$8x$
$17x + 5$
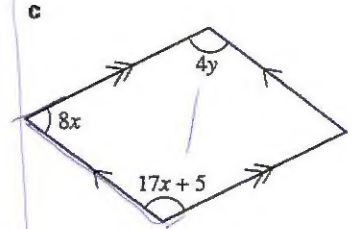
$25x + 5 = 180$
$x = 7°$

$17x + 5 = 4y$
$124 = 4y$
$31° = y$
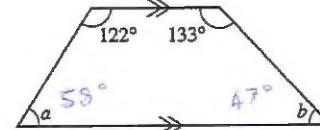
7 For each of these shapes, calculate the value of the lettered angles.

(Diagram: trapezium with angles 40° at top, 115°, a, b, 125° marked, 75°, 15°, 65° at bottom left, 40° at bottom right, c marked)

Handwritten: 40°, 115°, 75°, 65

Handwritten answers:
$a = 40°$
$b = 75°$
$c = 15°$

(Diagram: second shape with 75° i at top, d, g, 105°, j, 75°, 105°, h, f, e, 75°, 95°, 50°)
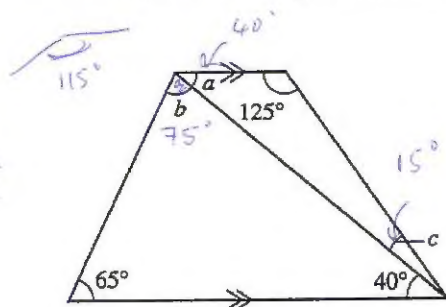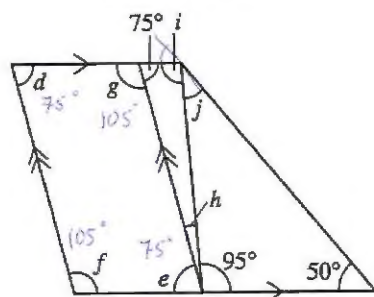
Handwritten answers:
$h = 180 - 95 - 75$
$= 10°$

$i = 180 - 75 - 10 = 95°$

$j = 180 - 95 - 50 = 35°$

8 Calculate the values of x and y in each of these trapeziums.
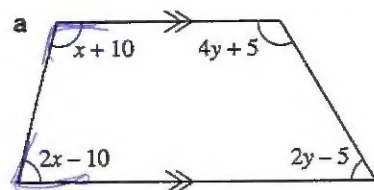
a

(Diagram: trapezium with $x + 10$, $4y + 5$ on top, $2x - 10$, $2y - 5$ on bottom)

Handwritten:
$3x = 180$
$x = 60°$
$6y = 180$
$y = 30°$

b

(Diagram: trapezium with $17x + 5$, $2y$ on top, $8x$, $3y - 20$ on bottom)

Handwritten:
$25x + 5 = 180$
$x = 7°$
$5y - 20 = 180$
$5y = 200$
$y = 40°$
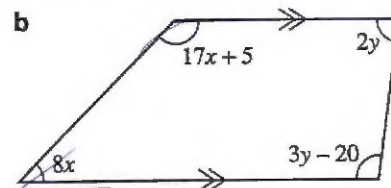
9 Calculate the value of x in each of these rhombuses.

a

(Diagram: rhombus with x, 25 at top, 25° at bottom left)

Handwritten: $x = 65$

b

(Diagram: rhombus with 34° at top, x, 34 at bottom)

Handwritten:
$\dfrac{180 - 34}{2} = 73°$

c

(Diagram: rhombus with 35, 55 at top, x, 55° and 35° at bottom)

Handwritten side note: 11, 25
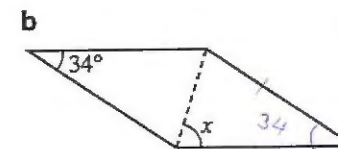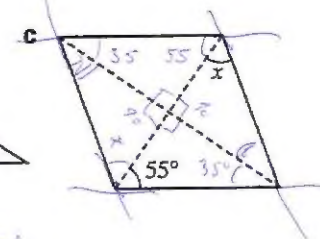
10 Calculate the values of the letters in each of these shapes.

a

(Diagram: shape with 15°, x, 140°, 25°, 25°, w, 65°, y, 90, 40°)

Handwritten:
$y = 180 - (90 + 25)$
$= 65°$

(small sketches 40)

b

(Diagram: trapezium with $5x$, y on top, x, 42° bottom left, 50° bottom right)
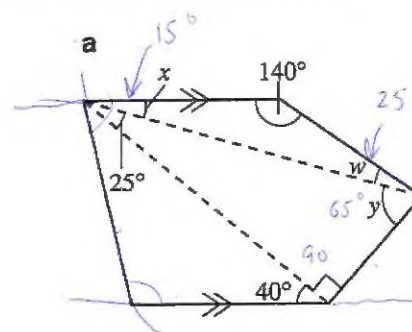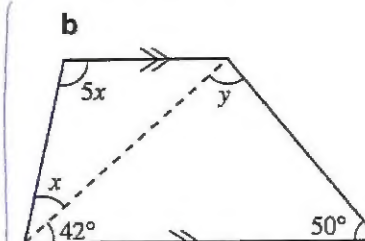
Handwritten:
$6x + 42 = 180$
$x = 23°$

$y = 180 - (50 + 42)$
$y = 88°$

Extension

11 Find the value of x in each of these quadrilaterals and hence state the type of quadrilateral it is.

  a One with angles $x + 10, x + 20, 2x + 20, 2x + 10$
  b One with angles $x - 10, 2x + 10, x - 10, 2x + 10$
  c One with angles $x - 10, 2x, 5x - 10, 5x - 10$
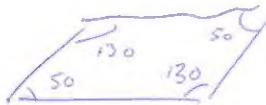 **d** One with angles $4x + 10, 5x - 10, 3x + 30, 2x + 50$

a.) $6x + 60 = 360$ $60°, 70°$
      $x = 50°$ $120°$ $110°$

   60 70
   120 110

TRAPEZIUM

b.) $6x = 360$ $50, 130$
      $x = 60°$ $50, 130$

   130 50
   50 130

PARALLELOGRAM OR RHOMBUS

c.) $13x - 30 = 360$ $20°, 60°, 140°, 140°$
      $x = 30°$

   60
   140 140 KITE
   20

d.) $14x + 80 = 360$ $90°, 90°, 90°, 90°$
      $x = 20°$

SQUARE OR
RECTANGLE

!PROOF **12**

A a 75
$105°$ B
$98°$ b 82
$100°$
d $80°$ $82°$ C
D
c
$98°$
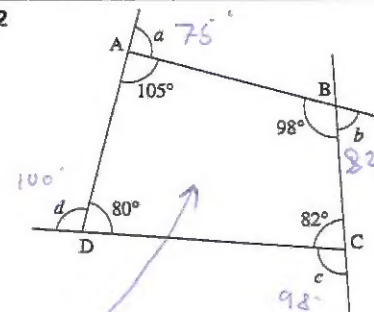
a The quadrilateral ABCD has interior angles 100°, 98°, 82° and 80°. Calculate the exterior angles (marked a, b, c, d) for each of the interior angles.
  What is the sum of the angles a, b, c, d?

b Prove that the sum of the exterior angles of any quadrilateral is 360°.

$a + b + c + d = 355°$

NOT POSSIBLE QUADRILATERAL